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Attenuating Pseudo S-waves in Acoustic Anisotropic Wave Propagation

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SUMMARY

The importance of anisotropy in seismic imaging has been recognized for several decades. In recent years, a growing number of anisotropic applications of reverse time migration (RTM) and full waveform inversion (FWI) have attempted to account for anisotropic effects of the real-world subsurface physics. In anisotropic media the P-wave, SV-wave, and SH-wave are intrinsically coupled, therefore the media are elastic in nature. However, the elastic anisotropic wave equation is computationally demanding and requires S-wave velocity model, for which in practice we often have little information. Consequently, the acoustic assumption is widely used in seeking anisotropic wave equations. Stable acoustic anisotropic wave equations may create apparent pseudo S-waves during wave simulation. S-wave contributions then show up as artifacts in migrated images from RTM and in velocity perturbations from FWI. We propose a model taper method to eliminate the source generated S-wave, and derive a series of approximate formulae that allow accurate and efficient attenuation of pseudo S-waves after acoustic anisotropic wave propagation, as a supplementary routine. Synthetic data and real data examples are shown to verify the proposed prescription in complex media.

Introduction

The anisotropic approximation to the elastic model was first introduced by Alkhalifah (1998), by setting the S-wave velocities to zero along the anisotropy axis of symmetry. The S-wave velocities can be still non-zero in other directions. This results in strong pseudo S-wave noise in wave propagation simulations. From Alkhalifah's dispersion relation, variants of coupled second-order acoustic anisotropic wave equations have been developed (e.g. Zhou et al., 2006; Du et al., 2008). These equations are kinematically equivalent but amplitude behavior may differ. More significantly, these equations suffer from instability in media of general inhomogeneity (Zhang et al., 2011). To overcome the problems of stability and pseudo S-waves, various authors derived a pure acoustic wave equation based on the P-wave dispersion relation (e.g. Liu et al., 2009; Chu et al., 2011; Xu and Zhou, 2014). The decoupled P-wave equation contains a pseudo-differential operator, which is difficult to tackle. For this reason, some sophisticated mathematical and numerical methods should be used to avoid alteration of wave propagation kinematics (e.g. Liu et al., 2009; Song and Fomel, 2010; Chu et al., 2011; Pestana et al., 2011; Xu and Zhou, 2014). There is no pseudo S-wave in this approach.

All equations derived from dispersion relations imply the assumption of locally constant media. Another way of obtaining pseudo acoustic wave equations starts from Hooke's law and the equations of motion (Duveneck and Bakker, 2011; Zhang et al, 2012). These wave equations have the advantage of being physically clear and straightforward to implement. Furthermore, their stability has been demonstrated (Zhang et al, 2011). We therefore prefer this kind of acoustic anisotropic wave equation. However there are still undesired S-waves present in the modeling. In this abstract, we first review stable acoustic anisotropic wave equations, and discuss the method of changing anisotropic parameters around the sources. Then we propose new equations for removing pseudo S-waves based on cancelling the S-wave component by operations on accurate simulated stresses. We calculate the pure P-wave from saved wavefields for each imaging time step for RTM or FWI.

Acoustic wave equations and Source-generated S-waves

We focus on second order acoustic wave equations for the principal stress vector $\boldsymbol{\sigma}=(\sigma_{xx},\sigma_{yy},\sigma_{zz})^T$. For simplicity, we assume density is constant. We discuss first vertical media, where the symmetry planes align with the coordinate axes. In vertical orthorhombic (VORT) media, the anisotropic wave equation is given by (Fowler and King, 2011; Zhang et al, 2012; Qin, 2014):

$$\frac{\partial^2\boldsymbol{\sigma}}{\partial t^2}=CD\boldsymbol{\sigma}+\boldsymbol{s},\qquad \text{where } D=diag\left(\partial_{xx}^2,\quad \partial_{yy}^2,\quad \partial_{zz}^2\right), \tag{1}$$

$\boldsymbol{s}$ is the source vector, and $C=\left(C_{ij}\right)$ denotes the symmetric 3x3 elastic matrix with velocity and six Thomsen parameters

$$\left(C_{ij}\right)=v_{p0}^2\begin{pmatrix} 1+2\varepsilon_2 & (1+2\varepsilon_2)\sqrt{1+2\delta_3} & \sqrt{1+2\delta_2} \\ (1+2\varepsilon_2)\sqrt{1+2\delta_3} & 1+2\varepsilon_1 & \sqrt{1+2\delta_1} \\ \sqrt{1+2\delta_2} & \sqrt{1+2\delta_1} & 1 \end{pmatrix}. \tag{2}$$

3D VTI media is a special case of VORT media with $\varepsilon_1=\varepsilon_2(=\varepsilon)$, $\delta_1=\delta_2(=\delta)$, $\delta_3=0$. In 2D VTI media, there is no term σ_{yy}, and the elastic matrix degenerates to a 2x2 matrix. We notice also in isotropic media the stress vector degenerates to the pressure wavefield and (1) becomes the isotropic acoustic wave equation.

For TI wave propagation, Duveneck and Bakker (2011) proposed to surround the source with a smooth tapered elliptically anisotropic region (i.e. $\varepsilon=\delta$) to suppress the source-generated S-wave. Although internal conversions between P- and S-waves always exist, this is a plausible and easy way to remove part of the S-wavefield while minimally influencing the kinematics. We generalize this method to orthorhombic media. We define an ellipsoidal region by

$$\varepsilon_1=\delta_1,\quad \varepsilon_2=\delta_2,\quad \delta_3=\left(\delta_1-\delta_2\right)/\left(1+2\delta_2\right), \tag{3}$$

and three zero angles. We can taper the medium around the source to be ellipsoidal to get rid of source-generated pseudo S-waves. The form of the source term was not discussed in Duveneck and Bakker (2011). We emphasize here that the components of the source vector are not identical. The source vector must be written as the following form

$$\boldsymbol{s} = \left(\sqrt{1+2\delta_2}\, f, \quad \sqrt{1+2\delta_1}\, f, \quad f\right)^T, \tag{4}$$

where f is a source signature. Marine surface acquisition automatically places the source in the isotropic water layer; our approach is useful for VSP, OBC and land data where the sources or receivers may be very near to or in strongly anisotropic media.

Figure 1 shows time snapshots in a VORT medium, $v_{p0} = 2000m/s$, $\varepsilon_1 = 0.3$, $\varepsilon_2 = 0.25$, $\delta_1 = 0.15$, $\delta_2 = 0.2$, $\delta_3 = -0.1$. Strong S-wave noise is present in (a), where the source is in the VORT medium. In (b)-(d), we modify the models in a region 150 m around the source. If the region is isotropic, quick changes in the tapers result in the artifact in (b); also a near 15 m horizontal wavefront delay can be identified, that leads to simulation traveltime error especially for FWI. If we taper the models in an ellipsoidal region, the snapshot (c) is nearly perfect; the horizontal traveltime error is negligible. In (d), the model is tapered as in (c), but the source vector components are identical.

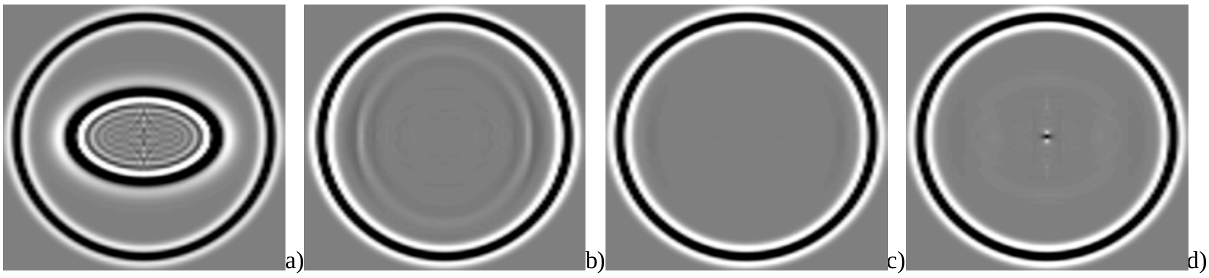


Figure 1 *VORT wavefield snapshots in homogeneous media. Crossline direction: (a) Without changes of models. Models are modified 150 m around the source in the three other cases; (b) Source in isotropic region; (c) Source in ellipsoidal region with the source term (4); (d) Source also in ellipsoidal region but with identical components in the source vector.*

Propagation S-wave attenuation

We follow the idea of Zhang and Zhang (2009) to express the pure P-wave by eigenvalue analysis. Assuming local homogeneity, equation (1) is rewritten in the wavenumber domain:

$$-\frac{\partial^2 \boldsymbol{\sigma}}{\partial t^2} = G\,\boldsymbol{\sigma}. \tag{5}$$

The transpose of the matrix G is given by

$$G^T = \begin{pmatrix} C_{11}k_x^2 & C_{12}k_x^2 & C_{13}k_x^2 \\ C_{12}k_y^2 & C_{22}k_y^2 & C_{23}k_y^2 \\ C_{13}k_z^2 & C_{23}k_z^2 & C_{33}k_z^2 \end{pmatrix}. \tag{6}$$

This transport matrix must have three non-negative eigenvalues λ_P, λ_{SV} and λ_{SH} corresponding to the squared phase velocities of three wave modes. Suppose the three corresponding linear independent eigenvectors are X_P, X_{SV} and X_{SH}; the matrix G can be diagonalized, and equation (5) becomes

$$-\frac{\partial^2}{\partial t^2}\begin{pmatrix} Y_P \\ Y_{SV} \\ Y_{SH} \end{pmatrix} = \begin{pmatrix} \lambda_P & 0 & 0 \\ 0 & \lambda_{SV} & 0 \\ 0 & 0 & \lambda_{SH} \end{pmatrix}\begin{pmatrix} Y_P \\ Y_{SV} \\ Y_{SH} \end{pmatrix}, \quad \text{with } \begin{pmatrix} Y_P \\ Y_{SV} \\ Y_{SH} \end{pmatrix} = \begin{pmatrix} X_P^T \\ X_{SV}^T \\ X_{SH}^T \end{pmatrix}\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \end{pmatrix}. \tag{7}$$

Then the three new wavefields Y_P, Y_{SV} and Y_{SH} are exactly the P-wave, SV-wave and SH-wave. Equations (7) imply that knowing the eigenvector of matrix G^T corresponding to the eigenvalue λ_P is the key for producing the pure P-wave component.

Under the acoustic assumption, the S-wave velocities must be very small in all directions $(\lambda_{SV} \approx \lambda_{SH} \approx 0)$. Employing this approximation, all three columns in matrix (6) can be considered as the eigenvectors to λ_P. We propose to take a weighted arithmetic average of the three columns:

$$Y_P = \frac{(c_x k_x^2)\sigma_{xx} + (c_y k_y^2)\sigma_{yy} + k_z^2\sigma_{zz}}{k_x^2 + k_y^2 + k_z^2}, \quad \text{with } c_x = \frac{C_{11}/C_{13} + C_{12}/C_{23} + C_{13}/C_{33}}{3},\ c_y = \frac{C_{12}/C_{13} + C_{22}/C_{23} + C_{23}/C_{33}}{3}. \tag{8}$$

For 2D VTI media, the coefficients in the equation are slightly different:

$$Y_P = \frac{\left(\dfrac{1+\varepsilon+\delta}{\sqrt{1+2\delta}}k_x^2\right)\sigma_{xx} + k_z^2\sigma_{zz}}{k_x^2 + k_z^2}. \tag{9}$$

Next we will consider more precise but costly formulae. The value of λ_P is given by the root of an algebraic equation of degree 3, we can consider its Taylor expansion with respect to anisotropic parameters (Zhang et al., 2012; Fowler and Lapilli, 2012). Our equations (8) and (9) are essentially the zero order approximation with reasonable compensation for higher order effects. To obtain better accuracy, we can use the correct first order approximation

$$\lambda_P \approx C_{11}k_x^2 + C_{22}k_y^2 + C_{33}k_z^2 - \frac{(C_{11}C_{22}-C_{12}^2)k_x^2k_y^2 + (C_{11}C_{33}-C_{13}^2)k_x^2k_z^2 + (C_{22}C_{33}-C_{23}^2)k_y^2k_z^2}{k_x^2+k_y^2+k_z^2}. \quad (10)$$

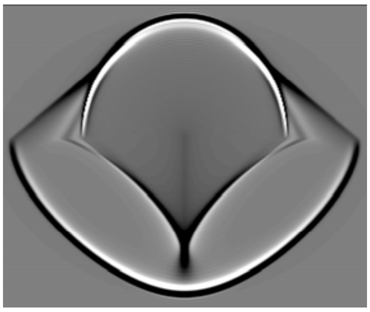
The eigenvector may be computed from the value in (10). The formula corresponding to (8) is of great complexity with many derivatives in VORT media, and is not very practical. But for VTI media, the formula becomes less complicated:

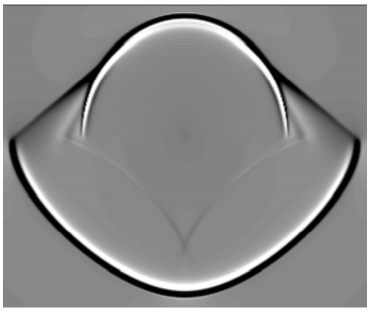
$$Y_P = \frac{P_{3D}\sigma_{xx} + Q_{3D}\sigma_{zz}}{(k_x^2+k_y^2+k_z^2)^2}, \text{ with } P_{3D} = \frac{(1+2\varepsilon)(3+4\varepsilon+2\delta)}{3\sqrt{1+2\delta}}(k_x^2+k_y^2)^2 + \sqrt{1+2\delta}(k_x^2+k_y^2)k_z^2, \, Q_{3D} = \frac{3+4\varepsilon+2\delta}{3}(k_x^2+k_y^2)k_z^2 + k_z^4. \quad (11)$$

For 2D VTI media,

$$Y_P = \frac{P_{2D}\sigma_{xx} + Q_{2D}\sigma_{zz}}{(k_x^2+k_z^2)^2}, \text{ with } P_{2D} = \frac{(1+2\varepsilon)(1+\varepsilon+\delta)}{\sqrt{1+2\delta}}k_x^4 + \sqrt{1+2\delta}k_x^2k_z^2, \; Q_{2D} = (1+\varepsilon+\delta)k_x^2k_z^2 + k_z^4. \quad (12)$$

To demonstrate the higher order P-wave construction formulae, we consider a 2D two-layer model. The first layer is isotropic with velocity equal 1500 m/s, and the second layer is VTI with $\varepsilon = 0.35$, $\delta = -0.1$, and velocity 2000 m/s. A point source is placed in the isotropic layer 25 m above the second layer. We do FD modeling using grid spacings $dx = dy = dz = 12.5m$. Figure 2 shows snapshots. Without S-wave attenuation, a diamond-shaped spurious SV-wavefront appears in (a). In (b), the residual S-wave is highly attenuated by equation (9), but weak pseudo S-waves remain. Applying the more precise equation (12), we obtain complete elimination of S-wave energy in (c).


a)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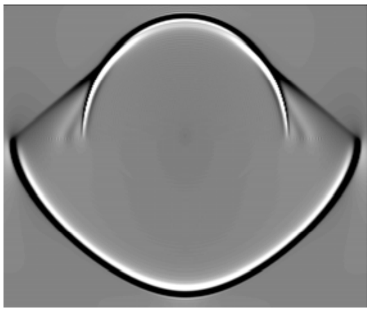
c)

Figure 2 *2D VTI wavefield snapshots for a two-layer model. (a) Without S-wave attenuation; (b) With S-wave attenuation using equation (9); (c) With S-wave attenuation using higher order equation (12).*

The acoustic wave equations in TTI media and tilted orthorhombic (TORT) media are computationally demanding (Duveneck and Bakker, 2011; Macesanu, 2011). To obtain more efficient equations, we can replace the derivatives in equation (1) by rotated differential operators, i.e. $(D_x, \; D_y, \; D_z)^T = R(\partial_x, \; \partial_y, \; \partial_z)^T$ where R is the transformation from local system to global system with 3 angles, and the adjoints of these operators, to yield simplified stable tilted media acoustic wave equations (Zhang et al., 2011; Zhang et al, 2012). Those wave equations give accurate amplitudes compared with the exact equations (Macesanu, 2011). All the above P-wave construction formulae (8), (9), (11), and (12) become valid for TTI and TORT media by rotating the wavenumbers.

Our formulae can be implemented by the finite-difference (FD) method, the pseudospectral method or a combined method in a mixed space-wavenumber domain. In order to solve the elliptic equations in the denominators, we prefer a FD implementation, which allows variable-length grids especially in the vertical direction. We have developed an approach based on a high-order FD scheme (Zhang et al, 2011). As an example, we show in Figure 3 TTI RTM images from a broadband survey in Angola. Using equation (8) and a high-order FD scheme, the P-wave is almost unchanged, and S-wave crosstalk, as nearly vertical artifacts on the image, is successfully removed.

Conclusions

The stable acoustic anisotropic wave equations have a major drawback in producing unwanted pseudo S-waves. Based on eigenvalue analysis, we present new formulae for attenuating those pseudo S-waves at each output step. We also extend from TI media to ORT media the method of modifying anisotropic parameters around the source, further attenuating source-generated S-waves. Our equations provide almost pure P-wave propagation.

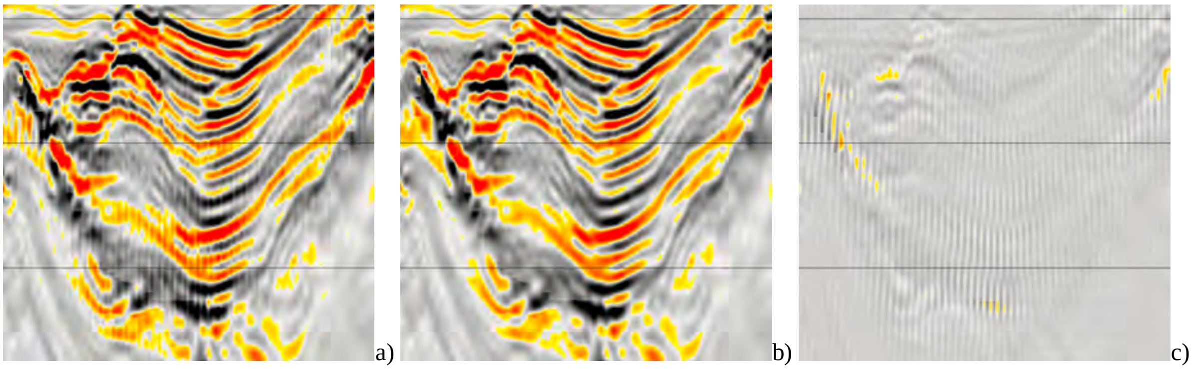

***Figure 3** Part of RTM images of BroadSeis real data with TTI models: (a)Without S-wave attenuation, pseudo S-wave related crosstalk is observed; (b)With S-wave attenuation; (c) The difference. The P-wave image is nearly untouched by S-wave attenuation.*

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