# Pre-stack wavelet estimation for broadband data

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## SUMMARY

Pre-stack elastic inversion requires a reliable wavelet model. This wavelet model must take into account the angle dependency of the wavelet, as well as the time dependency. This is even more important for broadband data, as the wavelets exhibit more variations, due to the fact that the maximum frequency is very high for small time and angles, but much smaller for large time and angle. We show how a continuously varying wavelet model can be estimated through a Bayesian inversion. This wavelet model can be used to pre-process the gathers and provide a zero-phase wavelet model for pre-stack elatic inversion. Synthetic and real data examples are shown to support this.


## INTRODUCTION

The angle-dependent reflectivity can be expressed by Shuey equation (1985):

$$R_P(t,\theta) = A(t) + B(t)\sin^2\theta \qquad (1)$$

with:

$$\begin{aligned} A &= \frac{1}{2}\left(\frac{\Delta V_P}{V_P} + \frac{\Delta\rho}{\rho}\right) \\ B &= \frac{\Delta V_P}{2V_P} - 4\left(\frac{V_S}{V_P}\right)^2\frac{\Delta V_S}{V_S} - 2\left(\frac{V_S}{V_P}\right)^2\frac{\Delta\rho}{\rho} \end{aligned} \qquad (2)$$

$A$ is the intercept term and $B$ the gradient term. We use the two-term equation, but our approach can be generalized to the three term Aki-Richards equation (1980).

In order to perform elastic inversion, we model the pre-stack angle gathers $D(t,\theta)$ by a convolutional model (* denotes convolution in $t$):

$$D(t,\theta) = W(t,\theta) * R_P(t,\theta) + N(t,\theta) \qquad (3)$$

where $W(t,\theta)$ is the pre-stack wavelet model, $N(t,\theta)$ the noise. In conventional narrowband data, the wavelet is weakly varying, as it is a Ricker-type wavelet with a dominant frequency decreasing with time and angle. For broadband data, the wavelet becomes strongly dependent on time and angle. This is because we attempt to push the maximum frequency $f_{max}(t,\theta)$ of the wavelet as far as possible: this means $f_{max}$ decrease quite fast with time and angle, due to both the stretch effect and the absorption.

There are two ways to estimate a pre-stack wavelet model. If a well with density logs $\rho$, P-velocity logs $V_p$ and S-velocity logs $V_s$ is available, $R_P$ can be computed for the well location and the wavelet estimated by determistic matching between $R_p$ and $D$. However this requires that the well is exactly at the position of the CIG, and that the checkshot allows a precise mapping. The wavelet is valid only at the vicinity of the well. Also, well are usually available on a small time window, which make it difficult to estimate the low frequencies of the wavelet. Even if the well is available on a large time window, it may be necessary to use a small window for the computation in order to ensure the stationarity of the wavelet.

If there are no wells, a zero-phase wavelet can be statistically estimated for a given time window and a given partial angle stack by assuming a white reflectivity. The problem here is that the autocorrelation of the noise corrupts the wavelet estimation and that no phase correction can be applied.

We consider both these methods as having limitations with broadband data and proceed to describe an algorithm that can estimate a wavelet model $W(t,\theta)$:

- having a continuous variation in $t$ and $\theta$
- having both an amplitude and a phase term
- not using local deterministic well information, but regional statistical well information

This last feature points to using Bayesian inversion, as described by Alemie and Sacchi (2011) who used Bayesian inversion to estimate the AVO parameters for a given wavelet model.

## A STATISTICAL MODEL FOR INTERCEPT AND GRADIENT BASED ON SOURCE SEPARATION

Intercept and Gradient have three known statistical features:

- there is an anti-correlation between $A$ and $B$ which is equivalent to state that "usually", $B$ and $A$ are of opposite sign, so that amplitude decreases with angle, while keeping the possibility of the "anomalic" case where $A$ and $B$ have the same sign and amplitude increases versus angle.
- reflectivities do not have a white spectrum but rather a blue spectrum and that they don't have a Gaussian distribution, but rather a "long-tail" distribution.

Well data can be used to extract these statistical properties in a quantitative way. From the $\rho, V_P, V_S$ well logs, $A_{well}, B_{well}$ can be computed from equation (2). We then perform a non-Gaussian source separation (Pham et al., 1992) on $A_{well}, B_{well}$. A source separation problem is $S = CE$:

$$s_i(t) = \sum_{j=1}^{N} c_{ij} e_j(t) \qquad (4)$$

where the N outputs $s_i(t)$ are known, and the correlation matrix $C = c_{ij}$ is unknown. In our case $N = 2$, $s_1(t) = A(t)$ and $s_2(t) = B(t)$. For a blind separation problem, the $e_j(t)$ are also unknown but are assumed to be independant with known probability density function (pdf) $p(x)$. If the pdf is gaussian, then only $V = C^T C$ can be recovered, and we can perform a singular value decomposition of the covariance to decompose into principal components. If the pdf is non Gaussian, then the matrix $C$ can be estimated. The maximum likelihood estimation of C can be done by estimating the separation matrix $D = C^{-1}$ such as the separated components computed by $F = DS$ are such as:

$$E[f_i(t)g[f_j(t)]] = \delta_{ij} \tag{5}$$

where $E$ design the expectation and $g(x) = -\frac{\partial}{\partial x} \log p(x)$. In the case of a gaussian distribution, $g(x) = x$ and we obtain that the covariance of the separated components must be the identity matrix.

Using this algorithm on a real well, we obtain the following separation matrix:

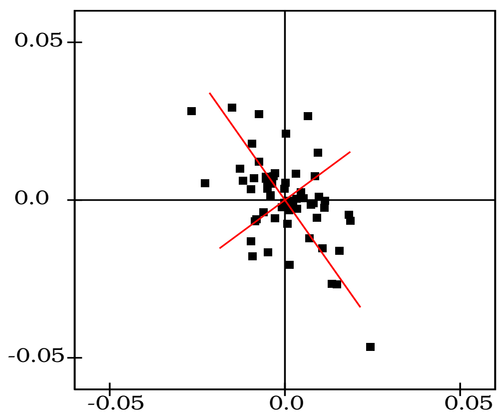


Fig.1: Well intercept-gradient crossplot and separation matrix.

We see that we have captured the anti-correlation of intercept and gradient.

We can after that whiten the separated components. We use a common wavelet for the two separated components, the amplitude spectrum of which is shown on Figure 2.

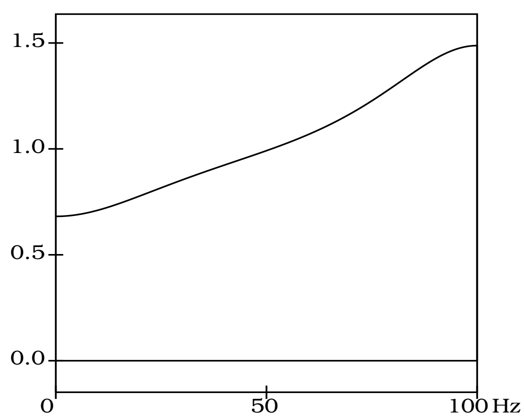


Fig. 2: Amplitude spectrum of the coloring wavelet.

We have at this point performed a convolutional source separation:

$$s_i(t) = \sum_{j=1}^{N} c_{ij}(t) * e_j(t) \tag{6}$$

The third item is the pdf of the whitened components. Again we use a common pdf shown in Figure 3. This pdf is a scaled logistic distribution, which has a tail in $e^{-|x|}$ but is a derivable function:

$$p(x) = \frac{1}{(2\cosh\frac{x}{2})^2} \tag{7}$$

We can see on Figure 3 that the logistic distribution (in red) has a longer tail than the gaussian distribution (in black), thus corresponding to a sparse input.

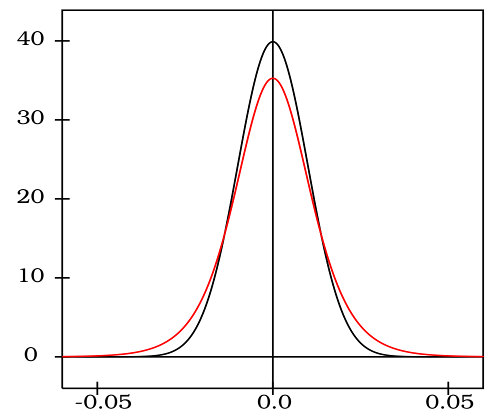


Fig. 3: Probability density function of the whitened separated components (red) and gaussian distribution (black).

## BAYESIAN PRE-STACK WAVELET INVERSION

Once the statistical properties of the intercept and gradient have been captured from well data, we can perform a Bayesian inversion for estimating the wavelets. The unknown of this inversion are:

- the whitened independent components $e_1(t)$ and $e_2(t)$
- the wavelet parameters $w_i$

From these unknowns, we compute:

- with equation (7) the intercept A(t) and gradient B(t)
- with Shuey equation (1) the angle dependent reflectivity $R(t,\theta)$
- with the $w_i$ parameters the angle dependent wavelets $W(t,\theta)$
- the modeled gather $D(t,\theta)$ by convolving the angle dependent wavelets and reflectivities

The wavelet model is an autoregressive moving average (ARMA) model with a separate phase-only term (all-pass) and amplitude-only term. This wavelet model is parametrically varying in time and angle. The Bayesian cost function is the sum of an a priori term and a data misfit term:

$$\begin{aligned} C &= \lambda \textstyle\sum_t f[e_1(t)] + f[e_2(t)] \\ &+ \textstyle\sum_{t,\theta} [D(t,\theta) - D_0(t,\theta)]^2 \end{aligned} \tag{8}$$

where:

- $D_0(t,\theta)$ is the actual gather given by the processing sequence
- the hyper-parameter $\lambda$ is the weight given to the a priori information compared to the data misfit term. Its theoretical value is twice the variance of the noise present in the data. In practice it can be chosen such as the two terms of the cost function have comparable magnitude after minimization.
- the function $f(x)$ is $f(x) = -\log p(x)$ where $p(x)$ is the probability density function of $e_1(t)$ and $e_2(t)$. For a logistic distribution $f(x) = 2\log\cosh x/2$ which behaves like $|x|$ for large $x$.

The first term in the cost function is the a priori term which is a constraint for $e_1(t)$ and $e_2(t)$ to be white independent processes with pdf $p(x)$, which is equivalent to constrain the desired statistical properties of the intercept and gradient (correlation, spectrum, sparseness). The second term is a constraint to match the data.

## SYNTHETIC DATA EXAMPLE

We have produced a intercept and gradient by generated two independent sources $e_1(t)$ and $e_2(t)$ following a logistic distribution (this can be done easily because if $y$ is a random variable uniform over $[0,1]$ then $x = \log \frac{y}{1-y}$ follows the logistic distribution given by equation (7)). The two independent sources have been mixed through the statistical model given by Figures 1 and 2, giving a intercept $A(t)$ and a gradient $B(t)$. Then an angle dependent reflectivity $R_0(t,\theta)$ was computed by equation (1) and is shown on Figure 5-b.

Figure 4-a shows an angle gather $D_0(t,\theta)$ produced by migrating with a wrong velocity the shot point produced by 1D modeling with the $R_0(t,\theta)$ model produced by the synthetic well. Comparing the angle gather $D_0$ with the reflectivity $R_0$, we see that the wavelet model must have a phase term to model the residual moveout (RMO) due to the wrong velocity and an amplitude term to model the stretch.

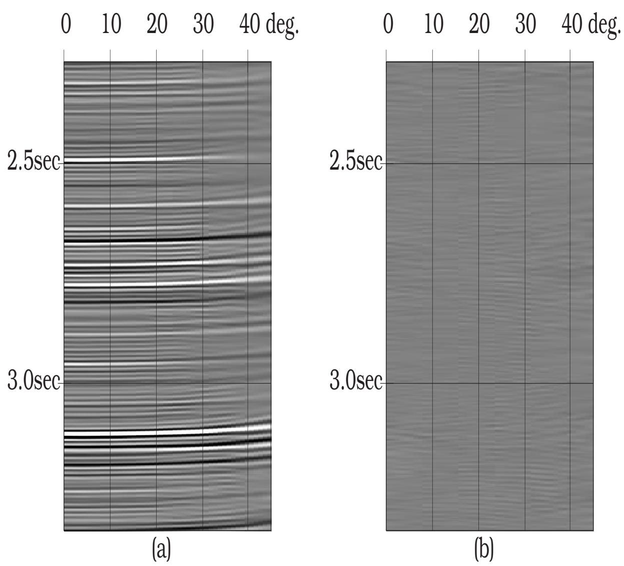


Fig. 4: Migrated gather (a) and data modeling error (b).

We use this gather as an input to our Bayesian inversion. Figure 4-b shows the data modeling error $D(t,\theta) - D_0(t,\theta)$ and Figure 5-a the derived $R(t,\theta)$ reflectivity gather together with the $R_0(t,\theta)$ produced by the well logs that was used to produce the gather (Figure 5-b).

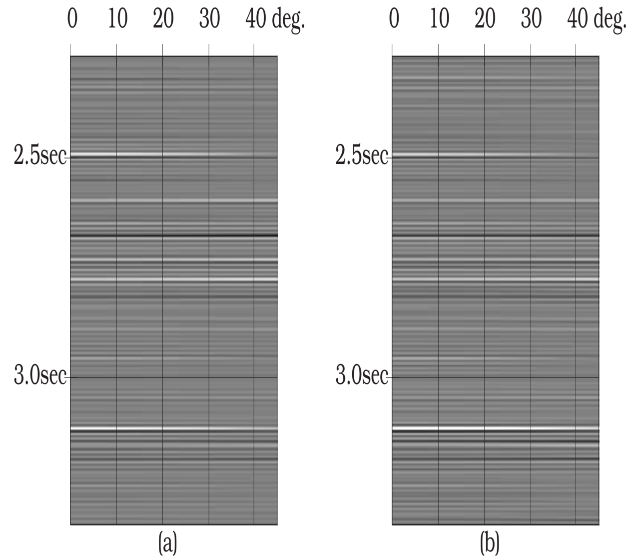


Fig. 5: Estimated (a) and well (b) synthetic reflectivity.

Figure 6 shows the wavelets estimated by the proposed Bayesian inversion as well as the wavelets estimated in non blind mode. In non blind (or deterministic) mode, the reflectivity $R(t,\theta)$ is constrainted to be the synthetic well reflectivity $R_0(t,\theta)$.

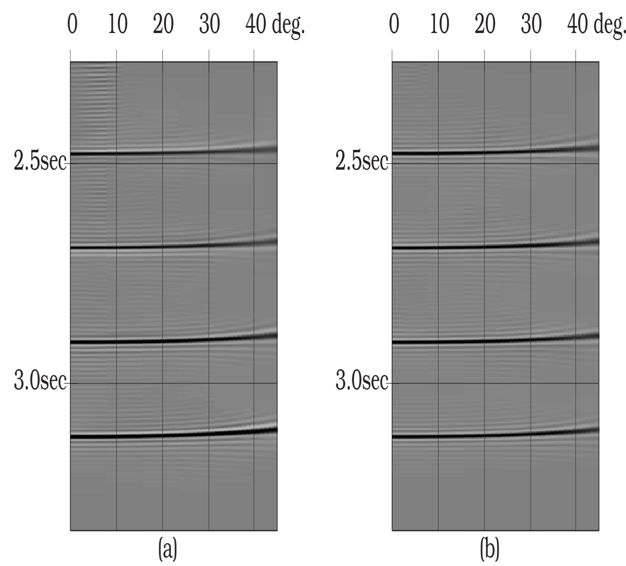


Fig. 6: Blind (a) and non-blind (b) wavelet model.

We can notice the similarity of the blind and non blind wavelet models. The blind inversion is able to capture the amplitude, phase, time and angle variation of the wavelet model.

## REAL DATA EXAMPLE

Figure 7-a shows an angle gather from a variable-depth streamer acquisition offshore North-West Australia and Figure 7-b the gather after noise attenuation and RMO, with the zero-phase

part of the wavelet model shown by the red curves in increments of 5 degrees. The RMO consists in removing from the data the phase-only part of the wavelet and the noise attenuation consists in replacing the data $D_0(t,\theta)$ by the reconstructed data $D(t,\theta)$:

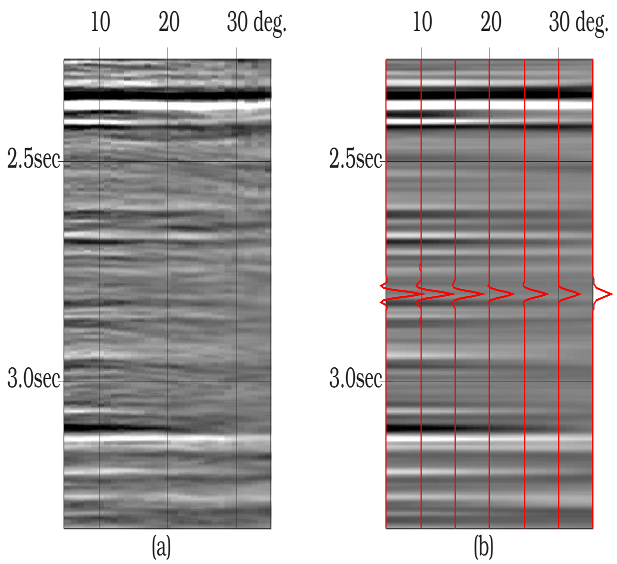


Fig. 7: Angle-gather (a) after RMO and noise attenuation (b).

Figure 8-a shows the removed noise $D_0(t,\theta) - D(t,\theta)$ and Figure 8-b shows the reflectivity $R(t,\theta)$:

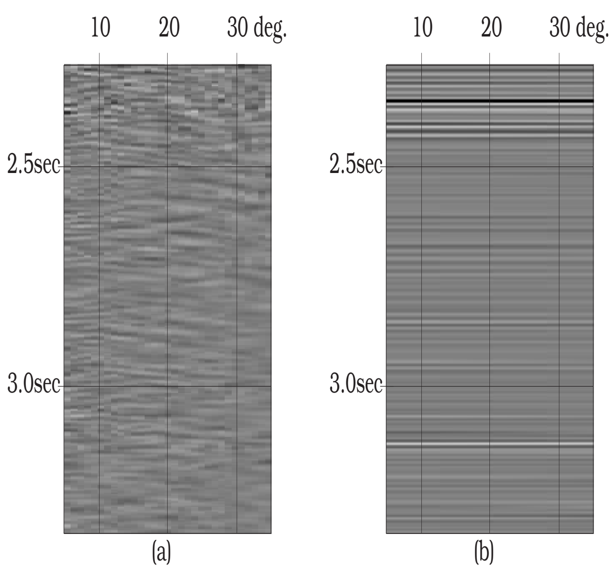


Fig. 8: Reconstruction noise (a) and reflectivity (b).

A pre-stack elastic inversion was performed with this dataset and the estimated wavelets. The impedance and $V_P/V_S$ sections are shown in Figure 9:

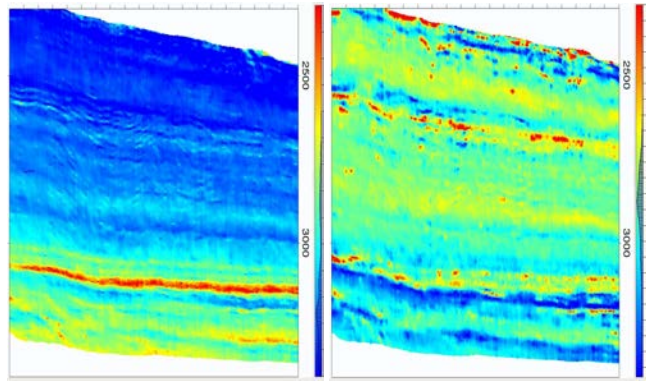

Fig. 9: $I_P$ (left) and $V_P/V_S$ (right) section with proposed method

This can be compared with Figure 10, showing the $I_p$ and $V_P/V_S$ sections obtained by using a deterministic angle-dependent wavelet model obtained by matching the well to the gather at the well location:

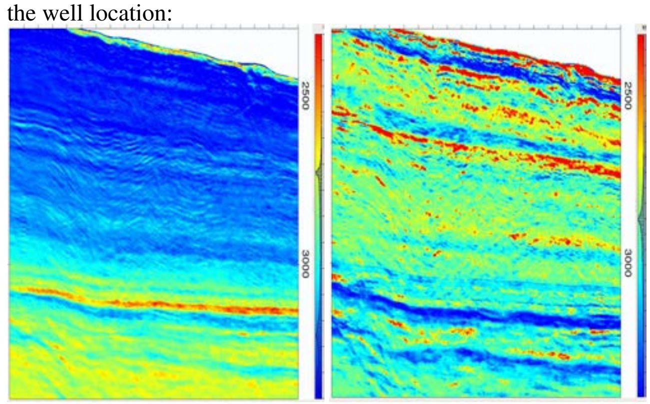

Fig. 10: $I_P$ (left) and $V_P/V_S$ (right) section with the conventional deterministic method

We see that the result with the proposed blind estimation wavelet model compares well with a deterministic model.

## CONCLUSION

We have described a Bayesian algorithm that can perform the preprocessing before elastic inversion (noise attenuation and residual move-out) as well as the angle-dependent zero-phase wavelet estimation. This algorithm uses the statistical properties of the well instead of the well itself. We have shown on a synthetic example that the statistical properties of the well allows to estimate a wavelet model that is close to the wavelet model obtained by deterministic matching. Using this method allows to model the large time and angle variations of the wavelet model of broadband data. As the statistical properties of the well are less localized than the well itself, the spatial variability of the wavelet can also be taken into account.

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